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The Use of Linear Solvers in Generalized Eigenvalue problems for Flow Instability Analysis.

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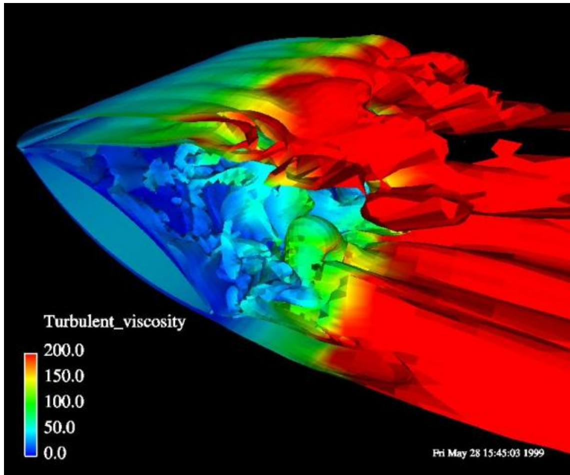
My questions

- Do I need a result for a customer or to publish a paper?
- How much time do I have?
- How much accuracy do I need?
- How much money can I spend? Do I have any?
- Do I have a cluster? How big?
- How is the geometry?
- Mesh dependance?

I will tell you about my life…….

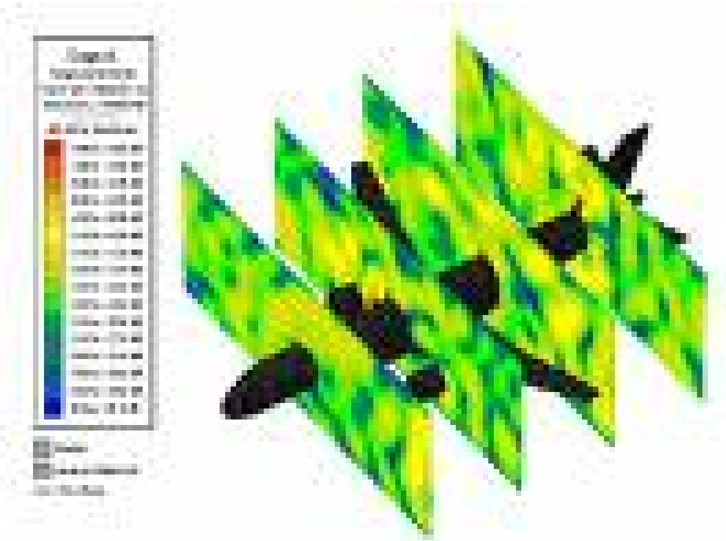
My computational life

- Step 1: Navier Stokes problems.
  - Fixed mesh/ Free surface problems
  - 2D/3D
  - Steady / Unsteady
  - Presence of 4 linear solvers per time step.
  - **Direct / Iterative Solvers**
  - Sequential/Paralell.



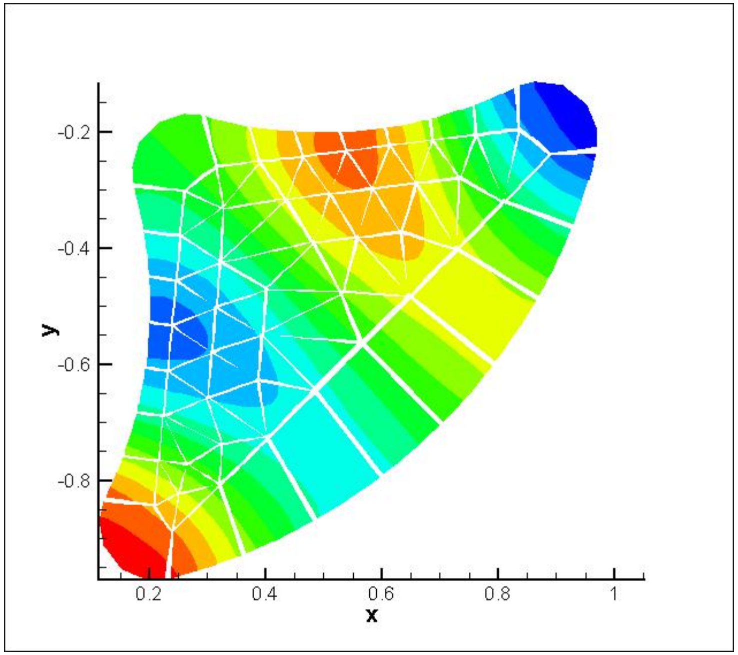
My computational life

- Step 2: Low frequency electromagnetism.
  - Fixed mesh
  - 2D/3D
  - Steady / frequency domain
  - **Direct / Iterative Solvers**
  - Sequential/Paralell.
  - Real/Complex matrices.



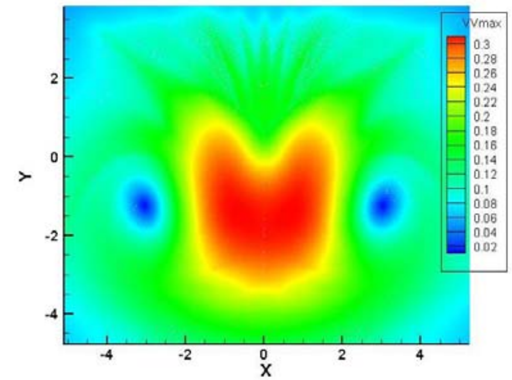
My computational life

- Step 3: Acoustics.
  - Fixed mesh
  - 2D/3D
  - Frequency domain
  - **Direct iterative/Iterative Solvers**
  - Real/Complex matrices.
  - PML boundary conditions.



My computational life

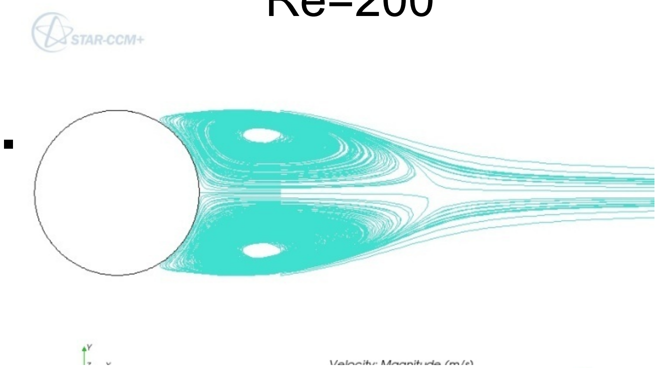
- Step 4: Smoothed Particle Hydrodynamics.



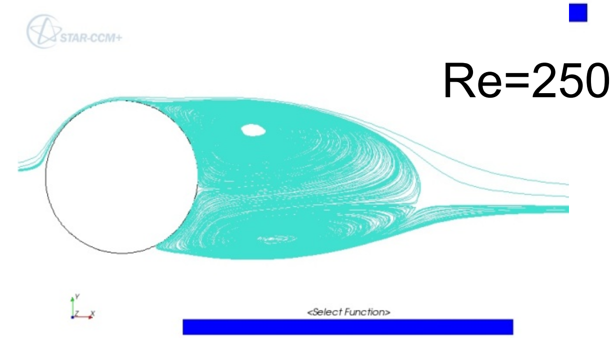
  - Lagrangian method.
  - 2D/3D
  - Time domain
  - **WCSPH/ISPH→Direct iterative/Iterative Solvers**
  - Sequential/Paralell/GPU.
  - Particles move every time step.

My computational life

Re=200



- Step 5: Linear flow Instability.



Re=250

  - Fixed mesh
  - 2D/3D
  - Steady / Unsteady
  - Presence linear solvers.
  - **Direct iterative/ Iterative Solvers(Sure?)**
  - Sequential/Paralell.
  - Time domain / frequency domain

Mathematical description.

Starting with the incompressible Navier-Stokes equations

Linearization around a particular 2D(x,y) steady Navier-Stokes solution: BASE FLOW

$$\bar{u}, \bar{v}, \bar{w}$$

$$u_i = \bar{u}_i + \tilde{u}_i \qquad p = \bar{p} + \tilde{p}$$

The perturbations follow the ansatz:

$$\tilde{u}_i = \widehat{u}_i(x,y)e^{i\beta z}e^{i\omega t}$$

$$\tilde{p} = \widehat{p}(x,y)e^{i\beta z}e^{i\omega t}$$

ω is Complex (Growth/Damping rate, Frequency) and β(wavenumber) is Real

Mathematical description.

Generalized Eigenvalue Problem

$$A\begin{pmatrix}\widehat{u}_1\\ \widehat{u}_2\\ \widehat{u}_3\\ \widehat{p}\end{pmatrix} = -i\omega B\begin{pmatrix}\widehat{u}_1\\ \widehat{u}_2\\ \widehat{u}_3\\ \widehat{p}\end{pmatrix}$$

A Complex matrix(4N x 4N)
B Real matrix(4N x 4N)

Generalized Eigenvalue Problem

$$A\begin{pmatrix}\widehat{u}_1\\ \widehat{u}_2\\ \widehat{u}_3\\ \widehat{p}\end{pmatrix} = -i\omega B\begin{pmatrix}\widehat{u}_1\\ \widehat{u}_2\\ \widehat{u}_3\\ \widehat{p}\end{pmatrix}$$

$$A = \begin{pmatrix}\alpha_{11} & \frac{\partial \overline{u}_1}{\partial y} & 0 & \frac{\partial}{\partial x}\\ \frac{\partial \overline{u}_2}{\partial x} & \alpha_{22} & 0 & \frac{\partial}{\partial y}\\ \frac{\partial \overline{u}_3}{\partial x} & \frac{\partial \overline{u}_3}{\partial y} & \alpha_{33} & i\beta\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & i\beta & 0\end{pmatrix}$$

$$\alpha_{ii} = \overline{u}_j\frac{\partial}{\partial x_j} + \frac{\partial \overline{u}_i}{\partial x_i} - \frac{1}{Re}(\frac{\partial^2}{\partial x_j^2} - \beta^2) + i\beta\overline{u}_3$$

$$B = \begin{pmatrix}M & 0 & 0 & 0\\ 0 & M & 0 & 0\\ 0 & 0 & M & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

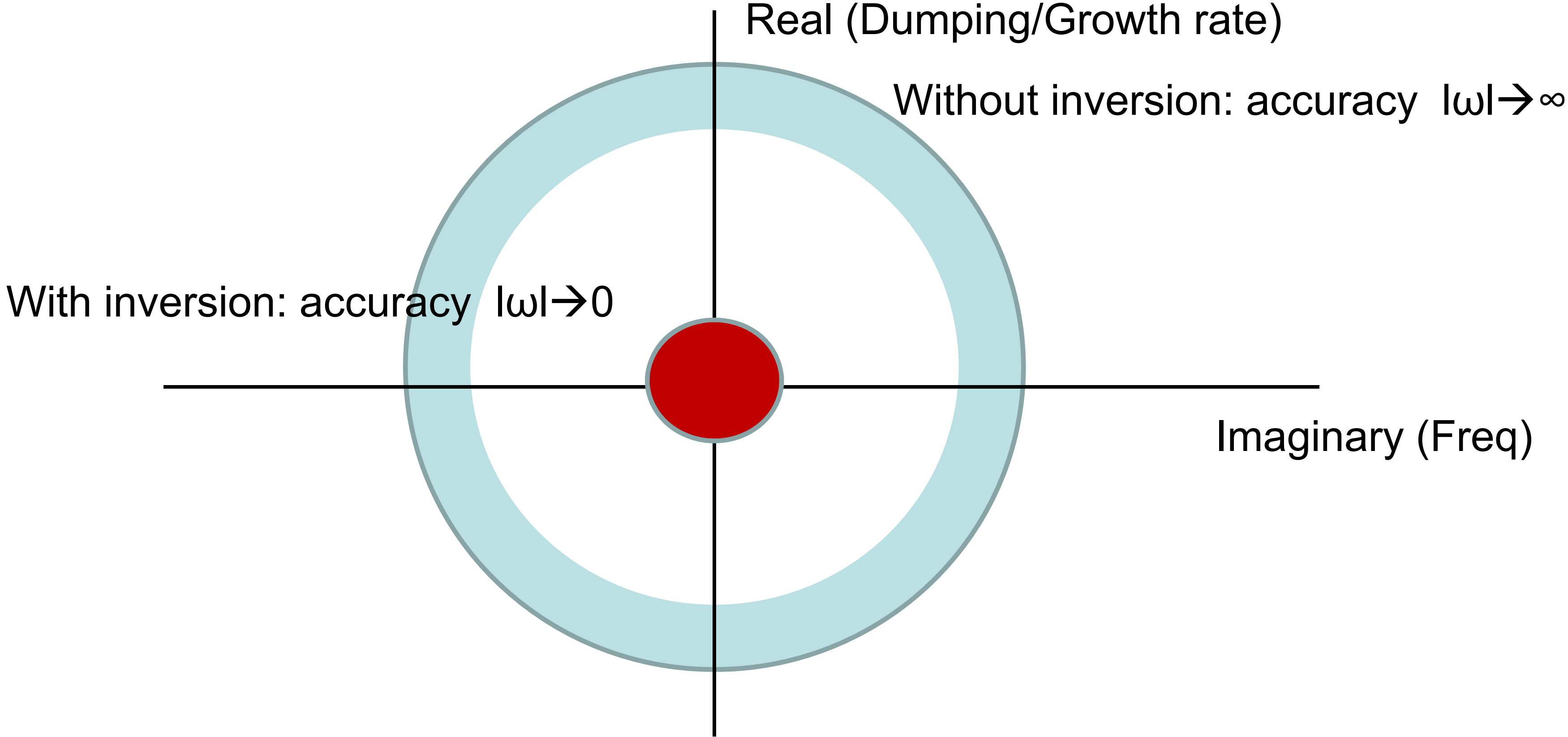
-B is non invertible, so we will invert A (un-symmetric matrix)
-If we are interested in the closest eigenvalues to the origin, we have to invert the problem

Singular problem: Shift and inverse strategy

Generalized Eigenvalue Problem

$$A\begin{pmatrix}\hat{u}_1\\ \hat{u}_2\\ \hat{u}_3\\ \hat{p}\end{pmatrix} = -i\omega B\begin{pmatrix}\hat{u}_1\\ \hat{u}_2\\ \hat{u}_3\\ \hat{p}\end{pmatrix} \quad A = \begin{pmatrix}\alpha_{11} & \frac{\partial \overline{u}_1}{\partial y} & 0 & \frac{\partial}{\partial x}\\ \frac{\partial \overline{u}_2}{\partial x} & \alpha_{22} & 0 & \frac{\partial}{\partial y}\\ \frac{\partial \overline{u}_3}{\partial x} & \frac{\partial \overline{u}_3}{\partial y} & \alpha_{33} & i\beta\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & i\beta & 0\end{pmatrix} \quad B = \begin{pmatrix}M & 0 & 0 & 0\\ 0 & M & 0 & 0\\ 0 & 0 & M & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

Why shift and inverse strategy?



Eigenvalue problems: Arnoldi method

m-Krylov iterative method.

Accuracy depends on **m.** (**m**=60÷100)

- **Very Sensitive Result**: LU **direct** solver used.
- I strongly recommend MUMPS.

$$A\begin{pmatrix}\hat{u}_1\\ \hat{u}_2\\ \hat{u}_3\\ \hat{p}\end{pmatrix} = -i\omega B\begin{pmatrix}\hat{u}_1\\ \hat{u}_2\\ \hat{u}_3\\ \hat{p}\end{pmatrix}$$

González Theofilis Gómez-Blanco AIAAJ 2007

Linear solvers: my experience

Taylor made solvers: precondiotioned CG and GMRES.

SparseKit (Iterative preconditioned GMRES solver)
How good must the preconditioner be? Tunning parameters

SuperLU (Good but …..)

MUMPS (Good job)

Have you used MUMPS?
(Jeff Crouch, Boeing, Seattle 2008)

Pouiseuille Flow (2D stability).

Analitic Base flow $U=1-y^2$

Orr-Sommerfeld validations.

Re=5772.22, 10k

$y \in [-1,1]$

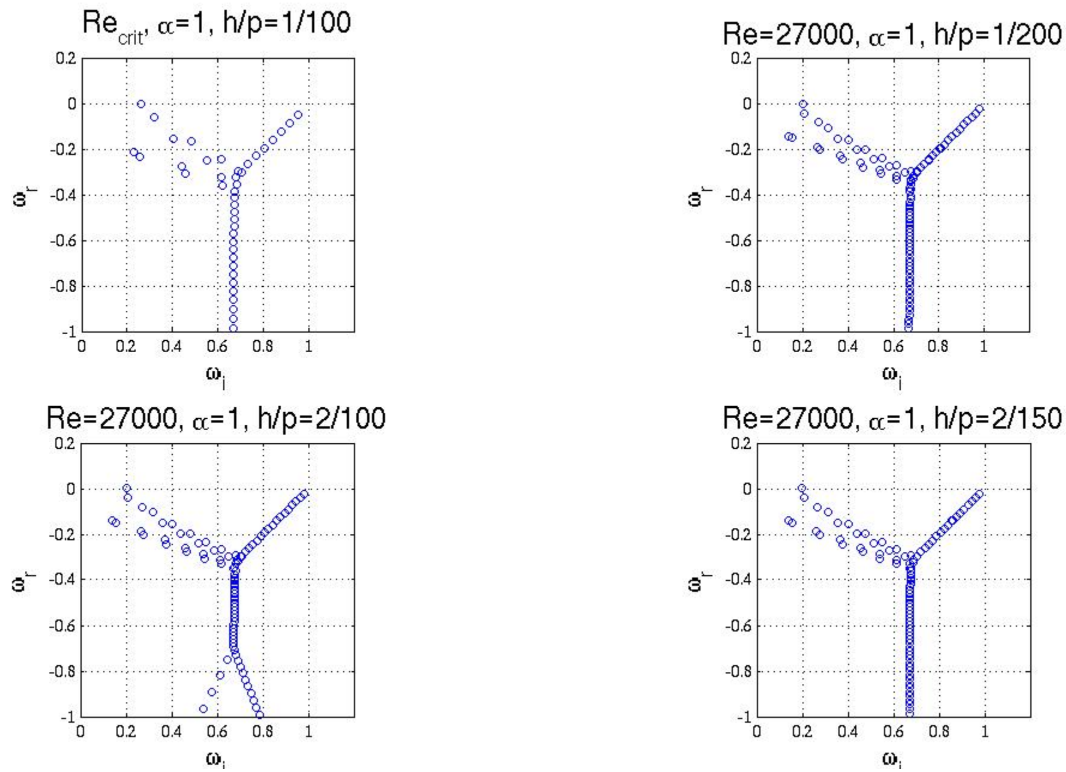


Table 1: Most unstable eigenvalue obtained at critical conditions, $Re = 5772.22, \alpha = 1.02056$, using different combinations of h and p. Reference result $\omega = (0.26400174, 5.9E-10)$[10].

h	p	ω_r	ω_i	h	p	ω_r	ω_i
1	30	0.2640118409	1.35E-5	2	30	0.2640017397	2.87E-9
1	40	0.2640017246	-4.36E-8	2	40	0.2640017395	3.02E-9
1	60	0.2640017395	3.02E-9	4	30	0.2640017395	3.02E-9
1	80	0.2640017395	3.02E-9	4	40	0.2640017395	3.02E-9

Orszag JFM 1971
Kirchner IJNMF 2000

Different Improvements

Duct Flow

Low order sparse Arnoldi SuperLU (p=2)

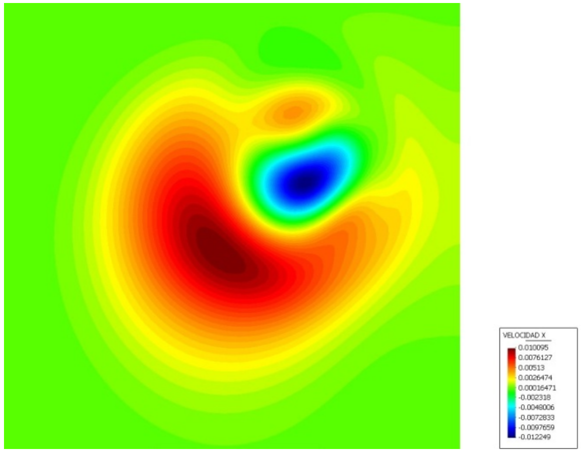
Re	nodes	Gb	time	Dumping rate	Freq.
100	60465	2.0	12 min	-0.140562	0.594190
1000	60465	3.1	24 min	-0.065256	0.858962

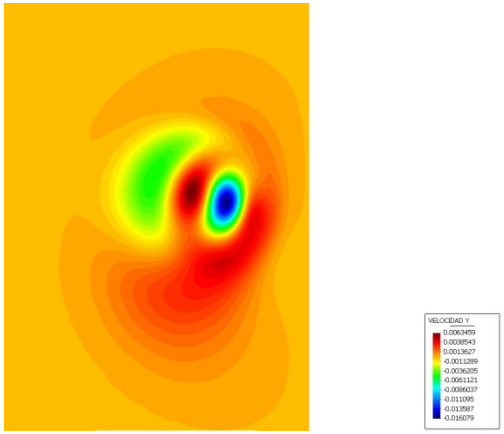
hp-FEM sparse Arnoldi MUMPS

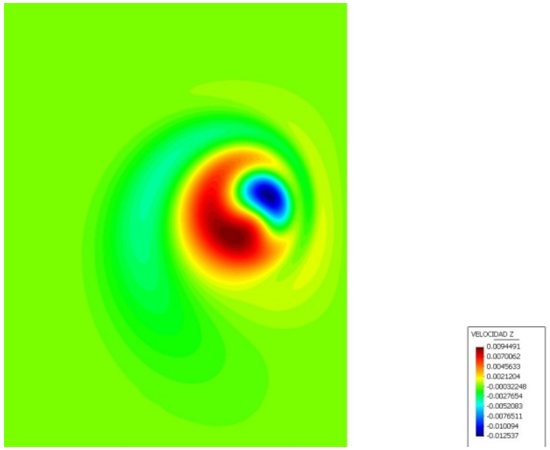
Re	h/p	Gb	time	Dumping rate	Freq.
100	1/10	0.6	8 sec	-0.140502	0.594178
1000	5/9	2.7	5 min	-0.065705	0.858584

Double vortex flow

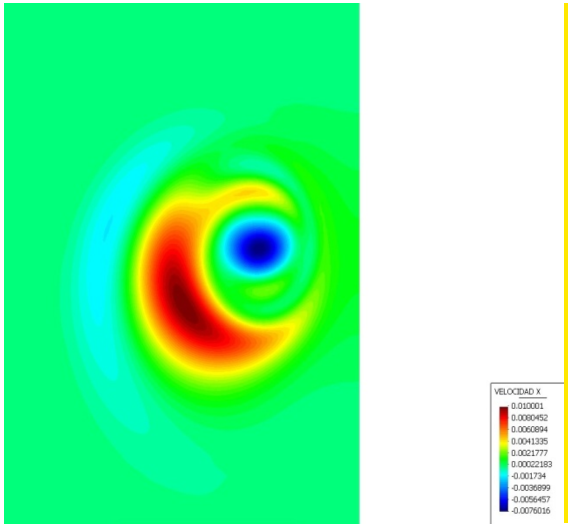
Double vortex dipole beta =3

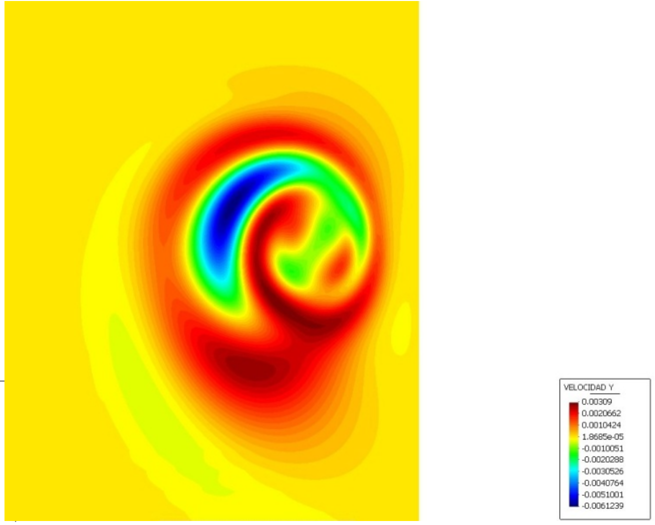


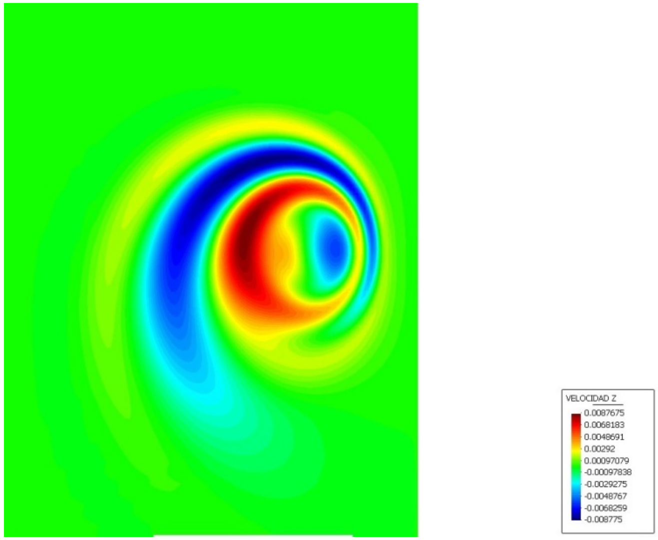




$\widehat{u}$ $\widehat{v}$ $\widehat{w}$







Gracias!

Specially for the MUMPS team.

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