Sparse factorizations using low rank submatrices

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Abstract

The paleoscientist Steven Jay Gould used “punctuated equilibrium” to describe the evolution of plants and animals over time. Change is not constant over time, but has plateaus of little change broken by short periods of rapid change.

This process holds for algorithm development also, with a prime example the solution of sparse linear systems. From Parter's 1960 paper outlining the connection between graph transformation and sparse factorization, to the minimum degree algorithm, to column-based SPARSPAK on vector computers, to supernode methods (like multifrontal) unlocking BLAS3 computations, to nested dissection for general graphs, and on to distributed memory systems, we have seen a series of algorithmic improvements that have significantly improved the state of the art.

I believe we are a threshold of another major improvement, at least for sparse linear systems that model some form of local coupling, e.g., the finite element or finite difference methods. One can consider the multifrontal tree generated by nested dissection to contain smaller sparse finite element matrices at the leaves, with the interior nodes of the tree containing dense boundary element matrices.

The past fifteen years have seen remarkable progress in solving boundary element linear systems, and we can use those techniques in the multifrontal method. Using a domain-decomposition of the subgraph of a separator to block the separator's matrix, we see that the dense off-diagonal submatrices are numerically rank deficient, and can be well-approximated using low rank matrices. There are great benefits to be gained, both in reducing factor storage and computations in both the factor and solve.

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