A Hybrid Iterative Solver Based on Block-Cimmino Method

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ABSTRACT

We study the solution of the system

$$Ax = b$$

where A is an $n \times n$ sparse matrix, x is an n-vector and b is an n-vector. We use the block Cimmino algorithm which involves subdividing the system into strips of rows, viz.

$$\begin{bmatrix} A_1 \\ \vdots \\ A_p \end{bmatrix} x = \begin{bmatrix} b_1 \\ \vdots \\ b_p \end{bmatrix}$$

and solving iteratively using

$$\begin{aligned} u_i &= A_i^+ b_i - P_{\mathcal{R}(A_i^T)} x^{(k)} \quad i = 1, \ldots .p \\ x^{(k+1)} &= x^{(k)} + \omega \sum_{i=1}^{p} u_i, \end{aligned}$$

where, A_i^+ is the Moore-Penrose pseudo-inverse and $P_{\mathcal{R}(A_i^T)} = A_i^+ A_i$ is the projector onto the range of A_i^T. The computation of the u_i is done by solving an augmented system generated from the partition A_i. This solve is done using the multifrontal direct solver `MUMPS`.

In this context, we study the convergence of the Block Cimmino method and how it can be improved. We look also at the parallel implementation and the scalability of the method on different problems. Finally, we compare these results with the multifrontal direct solver `MUMPS` and see where Block Cimmino can be a good alternative.

Keywords: sparse matrices, unsymmetric matrices, iterative methods, hybrid methods, parallel solver.
AMS(MOS) subject classifications: 65F05, 65F50.

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